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**Darren A Narayan\*** ([dansma@rit.edu](mailto:dansma@rit.edu)), School of Mathematical Sciences, 85 Lomb Memorial Drive, Rochester Institute of Technology, Rochester, NY 14623-5604. *Intermediate Ordered Colorings of Graphs.*

Given a graph  $G$ , a function  $f : V(G) \rightarrow \{1, 2, \dots, k\}$  is an ordered coloring or  $k$ -ranking of  $G$  if  $f(u) = f(v)$  implies every  $u - v$  path contains a vertex  $w$  such that  $f(w) > f(u)$ . A  $k$ -ranking is minimal if the reduction of any label greater than 1 violates the described ranking property. The rank number of a graph, denoted  $\chi_r(G)$ , is the minimum  $k$  such that  $G$  has a minimal  $k$ -ranking. The arank number of a graph, denoted  $\psi_r(G)$ , is the maximum  $k$  such that  $G$  has a minimal  $k$ -ranking. It was asked by Laskar, Pillone, Eyabi, and Jacob if there is a family of graphs where minimal  $k$ -rankings exist for all  $\chi_r(G) \leq k \leq \psi_r(G)$ . We given an affirmative response to their question showing that all intermediate minimal  $k$ -rankings exist for all paths, cycles, and  $K_{n_1, n_2, \dots, n_p}$  where  $n_{i+1} = n_i - 1$  for all  $1 \leq i \leq p - 1$ . (Received September 22, 2009)