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Gregory B Hurst* (ghurst2@illinois.edu), 808 Coventry Point, Springfield, IL 62702. *An elementary proof of Touchard's Congruence.*

The n th Bell number, denoted B_n , is the number of ways a set of n elements can be partitioned into nonempty subsets. It is easy to see that B_n is the sum of $S(n, k)$ where k ranges from 1 to n and $S(n, k)$ is the number of ways to partition a set of n elements into k nonempty subsets. We will consider a formula for the $n + j$ th Bell number which has just been discovered in the last two years. This formula states that B_{n+j} is the sum of $S(n, k)$ times a polynomial of degree j . This polynomial, denoted $P_j(k)$, also satisfies the recurrence relation $P_{j+1}(k) = P_j(k + 1) + kP_j(k)$ with base case $P_0(k) = 1$. Using this formula for B_{n+j} , relations such as Touchard's congruence:

$$B_{n+pr} \equiv B_{n+1} + rB_n \pmod{p}$$

where p is prime, can be proven elementarily. (Received September 19, 2009)