1056-11-197 **Ekin Ozman*** (ozman@math.wisc.edu). *What is a $\mathbb{Q}$ curve?*

A $\mathbb{Q}$-curve is an elliptic curve which is isogenous to all of its Galois conjugates. It is a mild generalization of an elliptic curve and has many interesting applications such as twisted Fermat type equations. A quadratic $\mathbb{Q}$-curve is a $\mathbb{Q}$-curve for which the smallest field of definition is a quadratic field. Quadratic $\mathbb{Q}$-curves of degree N defined over $\mathbb{K} = \mathbb{Q}(\sqrt{d})$ are parametrized by $X_0^d(N)$, the twist of $X_0(N)$ via w_N and the generator of the Galois group of $\mathbb{K}$ over $\mathbb{Q}$. Since cusps of $X_0(N)$ are rational it is immediate to say that $X_0(N)(\mathbb{Q})$ is non-empty. But w_N interchanges the cusps of $X_0(N)$ hence cuspidial points of the twist are not rational anymore. So it is not immediate to say if $X_0^d(N) = \emptyset$ or not. We will give an answer to the following question which is stated by Ellenberg:

For which $\mathbb{K}$ and N does $X_0^d(N)$ have rational points over every completion of $\mathbb{Q}$? (Received August 14, 2009)