Meeting: 1003, Atlanta, Georgia, AMS CP 1, AMS Contributed Paper Session

1003-11-838 **Curtis N Cooper*** (cnc8851@cmsu2.cmsu.edu), Dept. of Math. and Comp. Sci., Central Missouri State University, Warrensburg, MO 64093. *Factorizations of Some Periodic Linear Recurrence Systems.*

Let P and Q be relatively prime integers. The Lucas sequences are defined by $U_0 = 0$, $U_1 = 1$, $V_0 = 2$, $V_1 = P$, and

$$U_n = PU_{n-1} - QU_{n-2} \qquad \text{and} \qquad V_n = PV_{n-1} - QV_{n-2},$$

where $n \geq 2$. We will show that

$$U_n = \prod_{k=1}^{n-1}\Big(P - 2\sqrt{Q}\cos\frac{k\pi}{n}\Big), \quad n \geq 2.$$

and

$$V_n = \prod_{k=1}^{n}\Big(P - 2\sqrt{Q}\cos\frac{(k-\frac{1}{2})\pi}{n}\Big), \quad n \geq 1.$$

Next, let a_1, a_2, b_1, and b_2 be real numbers. The period two second order linear recurrence system is defined to be the sequence $f_0 = 1$, $f_1 = a_1$, and

$$\begin{aligned} f_{2n} &= a_2 f_{2n-1} + b_1 f_{2n-2} \\ \text{and} \quad f_{2n+1} &= a_1 f_{2n} + b_2 f_{2n-1} \end{aligned}$$

for $n \geq 1$. Also, let $D = a_1a_2 + b_1 + b_2$ and assume $D^2 - 4b_1b_2 \neq 0$. We will show that

$$f_{2n+1} = a_1 \prod_{k=1}^{n}\left(\frac{a_1+a_2}{2} \pm \sqrt{\Big(\frac{a_1-a_2}{2}\Big)^2 - b_1 - b_2 + 2\sqrt{b_1b_2}\cos\frac{k\pi}{n+1}}\right)$$

for $n \geq 0$.

The proofs will depend on finding the eigenvalues and eigenvectors of certain tridiagonal matrices. (Received September 30, 2004)