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1003-14-1526 **Dmitrii V Pasechnik***, Dept. E&OR, Tilburg University, P.O. Box 90153, 5000 LE Tilburg, Netherlands. *Hilbert's theorem on decomposability of nonnegative ternary forms as sums of squares of rational functions.* Preliminary report.

A partial answer to the 17th Hilbert problem was obtained by Hilbert himself, in a famous paper [H1], and in much less well-understood paper [H2].

In [H2] it is proved that every nonnegative ternary form is decomposable as a sum (at most 4, as was observed later) of squares of rational functions. It was used recently in [dKP] to develop an algorithm, based on semidefinite programming, to calculate such decompositions on computer, prompting a need to understand [H2] better.

We discuss an approach leading to a modern proof for [H2], at least in some important cases. It is related to the recent work [PRSS] that contains a modern proof of a theorem from [H1] on decomposability of nonneg. ternary quartics as a sum of 3 squares.

References.

[H1] D. Hilbert. Über die Darstellung definiten Formen als Summen von Formenquadraten, Math. Annalen, 32(1888), 342-350.

[H2] D. Hilbert. Über ternäre definite Formen, Acta Math. 17(1893), 169-198

[dKP] E. de Klerk and D.V. Pasechnik. Products of positive forms, linear matrix inequalities, and Hilbert 17-th problem for ternary forms, European J. OR, 157(2004) 39-45

[PRSS] V. Powers, B. Reznick, C. Scheiderer, and F. Sottile. Comptes Rendus, to appear. math.AG/0405475. (Received October 05, 2004)