Meeting: 1003, Atlanta, Georgia, SS 24A, AMS Special Session on Design Theory and Graph Theory, I

1003-15-315 **Jason J Molitierno*** (`molitiernoj@sacredheart.edu`), Department of Mathematics, Sacred Heart University, 5151 Park Avenue, Fairfield, CT 06825-1000. *The Algebraic Connectivity of Planar Graphs.* Preliminary report.

For any graph on n vertices whose vertices are labelled $1, \ldots, n$, we can associate the Laplacian matrix which is the $n \times n$ matrix $L = (\ell_{i,j})$ where

$$\ell_{i,j} = \begin{cases} -1, & \text{if } i \neq j \text{ and } i \text{ is adjacent to } j, \\ 0, & \text{if } i \neq j \text{ and } i \text{ is not adjacent to } j, \\ d_i, & \text{if } i = j \end{cases}$$

where d_i is the degree of vertex i. It is known that the Laplacian matrix is positive semidefinite. Thus the eigenvalues can be arranged in a nondescending order: $0 = \lambda_1 \leq \lambda_2 \leq \ldots \leq \lambda_n$. The eigenvalue λ_2 is known as the algebraic connectivity of a graph. Some known results for λ_2 are: (a) $\lambda_2 > 0$ if and only if the graph is connected, (b) given a graph, adding edges between nonadjacent vertices causes the algebraic connectivity to monotonically increase, and (c) if d is the minimal degree of a vertex in a noncomplete graph, then $\lambda_2 \leq d$. In a planar graph, it is known that $d \leq 5$. Thus for any planar graph, $\lambda_2 \leq 5$. In this talk, I show that we can sharpen the upper bound to be 4. We also show planar graphs such that $\lambda_2 = 4$. (Received September 09, 2004)