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1003-26-1689 **Charles N. Delzell*** (`delzell@math.lsu.edu`), Department of Mathematics, Louisiana State University, Baton Rouge, LA 70803. *Extension of the Fourier-Budan theorem to one-variable signomials.* Preliminary report.

Let $f(x) = a_0x^{r_0} + a_1x^{r_1} + \cdots + a_kx^{r_k}$, where each $a_i \in \mathbb{R}$, each $r_i \in \mathbb{N} := \{0, 1, \ldots\}$, and $r_0 < r_1 < \cdots < r_k$. Suppose $u < v$. Let $z(f, u, v) =$ the number of roots of f in $(u, v]$, counted with multiplicity. For any $w \in \mathbb{R}$ and $n \in \mathbb{N}$, let $s(f, w, n) =$ the number of sign-changes in the sequence $f(w), f'(w), f''(w), \ldots, f^{(n)}(w)$ (skipping over zeros). Then the Fourier-Budan Theorem says that $z(f, u, v) \leq s(f, u, r_k) - s(f, v, r_k)$ and $z(f, u, v) \equiv s(f, u, r_k) - s(f, v, r_k) \pmod 2$. In this paper we weaken the hypothesis of this theorem by allowing the r_i to be arbitrary real numbers; but we must then restrict u and v to be positive, to avoid non-real values of $f(x)$. Our conclusion is then that there exists $N \in \mathbb{N}$ such that for all $n \geq N$, $z(f, u, v) \leq s(f, u, n) - s(f, v, n)$ and $z(f, u, v) \equiv s(f, u, n) - s(f, v, n) \pmod 2$. We give an explicit upper bound on such an N, in terms of v and the a_i and r_i. The main idea of the proof is to let i_0 be the *least* $i \in \{0, 1, \ldots, k\}$ (if any) such that $r_i \notin \mathbb{N}$, and then to show that for large n, $f^{(n)}(v) \cdot \frac{d^n}{dx^n} a_{i_0} x^{r_{i_0}}|_{x=v} > 0$. We also show that such an extension is impossible for arbitrary real analytic f. (Received October 07, 2004)