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Lászlo Lovász*, Microsoft Research, 1 Microsoft Way, Redmond, WA 98052. *Graph limits and graph homomorphisms.*

Is there a notion of a limit of a growing graph sequence? What kind of object is this limit? Which parameters of graphs behave "continuously" when passing to the limit?

Limits of graph sequences can be defined in more than one setting. The case when the graphs in question are dense is best understood, but a lot of work has been done on the limit of growing sequences of graphs with bounded degree; in fact the limit object was defined first in this setting by Aldous (for sequences of trees) and by Benjamini and Schramm (for the general case).

In both cases, the key notion is that of a convergent graph sequence: a sequence of graphs is convergent if for every fixed graph F , the number of homomorphisms of F into the members of the sequence, after appropriate normalization, converges to a limit. If this holds, a limit object exists; in the case of dense graphs, one way of describing it is a symmetric measurable function $[0, 1]^2 \rightarrow [0, 1]$. Other cryptomorphic descriptions include graph parameters that are "reflection positive", and random graph models satisfying certain natural conditions.

Counting homomorphisms between graphs has a surprising number of applications. Many models in statistical mechanics and many questions in extremal graph theory can be phrased in these terms; as a consequence, the limiting values of many important graph parameters can be read off from limit object. This is closely related to "property testing" in computer science.

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