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**Curtis Cooper\*** (cnc8851@cmsu2.cmsu.edu), Dept. of Math. & Comp. Sci., Central Missouri State University, Warrensburg, MO 64093. *Bounds on a Remainder Term Associated with the Number of Base  $q$  Digits  $\geq d$  Function.*

Let  $q \geq 2$  be a fixed integer. The base  $q$  representation of a positive integer  $k$  can be written in the form

$$k = \sum_{r=0}^{\infty} a_r(q, k)q^r, \text{ where } a_r(q, k) \in \{0, 1, \dots, q-1\}.$$

Let  $d$  be a nonzero base  $q$  digit. Define the ‘number of base  $q$  digits  $\geq d$ ’ function as

$$\alpha_{\geq d}(q, k) = \sum_{r=0}^{\infty} [a_r(q, k) \geq d].$$

Here, we use Iverson’s notation of putting brackets around a true-false statement. A bracketed true-false statement is 1 if the statement is true and 0 if it is false. For an integer  $n \geq 1$ , let

$$A_{\geq d}(q, n) = \sum_{k=1}^{n-1} \alpha_{\geq d}(q, k).$$

First, we will show that

$$A_{\geq d}(q, n) = \left(1 - \frac{d}{q}\right)n \log_q n + O(n).$$

Second, we define

$$S_{\geq d}(q, n) = A_{\geq d}(q, n) - \left(1 - \frac{d}{q}\right)n \lfloor \log_q n \rfloor$$

where  $\log_q$  denote the logarithm function with base  $q$  and  $\lfloor \cdot \rfloor$  denotes the greatest integer function. We then show that if

$$c > \max \left\{ \frac{d}{q}, 1 - \frac{d}{q} \right\},$$

then

$$-c < \frac{S_{\geq d}(q, n)}{n} < 1 - \frac{d}{q}.$$

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