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Christopher Rasmussen* (crasmus@rice.edu), Department of Mathematics – MS 136, Rice University, P. O. Box 1892, Houston, TX 77251-1892. *A finiteness conjecture for abelian varieties over number fields.*

Let K be a number field. For a prime ℓ , let ζ_ℓ be a primitive ℓ -th root of unity. Let \tilde{K}_ℓ be the maximal pro- ℓ extension of $K(\zeta_\ell)$ unramified outside ℓ . Consider the following question: How many K -isomorphism classes of abelian varieties A of dimension g are there with the property that for some ℓ the ℓ -power torsion of A is rational over \tilde{K}_ℓ ? The conjecture is that this set of isomorphism classes is finite for fixed K and g .

This question is related to the arithmetic of Galois coverings of \mathbb{P}^1 minus 3 points. When such covers have degree a power of ℓ , the covering curves often have Jacobians whose ℓ -power torsion lies in an interesting subfield $\Omega_\ell \subseteq \tilde{K}_\ell$. The interaction of geometry and arithmetic on such curves is quite interesting.

We discuss the current status of the conjecture, which has been proven in certain cases. For the case $g = 1$, we give a precise list of those elliptic curves which have the above property. We also describe consequences of this conjecture with the study of coverings and a long-standing open question of Ihara.

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