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Mikhail Kapranov*, Yale University. *Riemann-Roch for determinantal gerbes and infinite-dimensional bundles.*

Let $\pi : \Sigma \rightarrow B$ be a smooth circle fibration over a base B and E be a complex vector bundle on Σ . Then $p_*(E)$, the L^2 -direct image of E is an infinite-dimensional bundle on B . This bundle has a $GL(\infty)$ -structure where $GL(\infty)$ is the “Japanese” group of automorphisms of a polarized Hilbert space. The determinantal central extension of $GL(\infty)$ gives rise to the determinantal class $C_1(p_*(E)) \in H^3(B, \mathbf{Z})$.

An interesting example is when $B = C^\infty(S^1, X)$ is the free loop space of an algebraic variety X , when $\Sigma = S^1 \times B$ and E is the pullback of the tangent bundle on X under the evaluation map $\Sigma \rightarrow X$. In this case $p_*(E)$ is the tangent bundle of B .

A local Riemann-Roch theorem established in a joint work with E. Vasserot in an algebro-geometric situation, describes $C_1(p_*(E))$ as a direct image of a characteristic class on E . In the example above this explains the results of Gorbounov, Malikov and Schechtman on the gerbes of chiral differential operators.

The talk will also discuss a more general “real Riemann-Roch” theorem for a family of real compact manifolds of any dimension d . This theorem is the subject of a work in progress joint with P. Bressler, B. Tsygan and E. Vasserot. (Received April 25, 2005)