1014-34-824 **Kwang C. Shin*** (`kcshin@math.missouri.edu`), Department of Mathematics, University of Missouri, Columbia, MO 65211. *Eigenvalues of non-self-adjoint Schrödinger operators with polynomial potentials.* Preliminary report.

For integers $m \geq 3$, the Schrödinger eigenvalue problem

$$-\frac{d^2}{dx^2}u(\lambda, x) + (x^m + P(x))u(\lambda, x) = \lambda u(\lambda, x), \quad x \geq 0, \tag{1}$$

will be studied, with Dirichlet or Neumann or Robin boundary condition at x=0, and $u(\lambda, +\infty) = 0$, where P is a polynomial of degree $\leq m-1$. This includes interesting special cases when all coefficients of P are real. Sibuya showed that the eigenvalues of this boundary value problem are the zeros of an entire function $E(\lambda)$ of order $1/2 + 1/m$.

In this talk, we present results on asymptotic expansions of $E(\lambda)$, eigenvalue asymptotics, and inverse spectral problems. Also we will mention interesting results for the full-line problem and for some non-standard problems as well. For example, the eigenvalues of non-self-adjoint $\mathcal{PT}$-symmetric anharmonic oscillators are all real and positive with, at most, finitely many exceptions.

Bender and Wu, Maslov, Sibuya, Birman and Solomyak, Helffer and Robert, and many others obtained asymptotic expansions of the eigenvalues with particular choices of the polynomial P or the boundary condition. (Received September 25, 2005)