

1014-97-1272 **Michael Livshits*** (michaelliv@gmail.com), 36 Linnaean Street, Apt. #14, Cambridge, MA
02138. *Algebra and Uniform Estimates Give a Face-lift to Calculus and Introductory Analysis.*

Calculus of polynomials is easy because $x - a$ divides $p(x) - p(a)$. As Hermann Weyl wrote in "Classical Groups" (p.4), a continuous f is differentiable at a if $x - a$ divides $f(x) - f(a)$ in the ring of continuous functions. This point of view is close to the original Weierstrass' definition.

To generalize, consider an algebra A of *nice* functions. We call a nice function f *nicely differentiable* if $f(x) - f(a) = q_a(x)(x - a)$ for some q_a in A and if $f'(x) = q_x(x)$ is also nice. All the formal aspects of Calculus become obvious.

By taking the Lipschitz (but any modulus of continuity will do) functions as nice, we get the uniform estimate $|f(x) - f(a) - (x - a)f'(a)| \leq K(x - a)^2$ that becomes our definition of the derivative. The *Monotonicity Theorem*, saying that f is increasing if $f' \geq 0$, is the hub of a streamlined version of Calculus that is practical, but may be more accessible to HS and college students than the usual one, and is closer to the modern mathematical ideas. It also can be a stepping stone to more advanced aspects of analysis. This approach is due to Hermann Karcher.

See details on world.std.com/~michaell/calculus.html

and math.uni-bonn.de/people/karcher/Reformen.html (Received September 27, 2005)