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Michael Z. Spivey* (mspivey@ups.edu), Department of Math and Computer Science, University of Puget Sound, Tacoma, WA 98416. *Combinatorial Sums via Finite Differences.*

We present a new approach to evaluating certain combinatorial sums by using finite differences. Let $\{a_k\}_{k=0}^{\infty}$ and $\{b_k\}_{k=0}^{\infty}$ be sequences with the property that $\Delta b_k = a_k$ for $k \geq 0$. Let $G(n) = \sum_{k=0}^n \binom{n}{k} a_k$, and let $H(n) = \sum_{k=0}^n \binom{n}{k} b_k$. We derive an expression for $G(n)$ in terms of $H(n)$ and for $H(n)$ in terms of $G(n)$. These expressions allow certain kinds of binomial sums to be evaluated fairly easily. We then extend our approach to handle binomial sums of the form $\sum_{k=0}^n (-1)^k \binom{n}{k} a_k$, $\sum_k \binom{n}{2k} a_k$, and $\sum_k \binom{n}{2k+1} a_k$, as well as sums involving unsigned and signed Stirling numbers of the first kind, $\sum_{k=0}^n \left\{ \begin{matrix} n \\ k \end{matrix} \right\} a_k$ and $\sum_{k=0}^n s(n, k) a_k$. (Received September 04, 2006)