1023-13-959

Greg G Oman^{*} (oman@math.ohio-state.edu), 5011 Godown Rd. Apt. D, Columbus, OH 43220. Some Results on Jónsson Modules over Commutative Rings with Identity.

Let M be an infinite unitary module over a commutative ring R with identity. M is called a Jónsson module provided every proper submodule of M has smaller cardinality than M. Robert Gilmer and Bill Heinzer studied this and related notions in several earlier papers. We build on some of their results and give a treatment of other questions not addressed in their papers. In particular, we present a very short proof of the result of W.R. Scott that if an infinite abelian group G is such that all proper subgroups of G have smaller cardinality than G, then $G \cong \mathbb{Z}(p^{\infty})$ for some prime p. We classify the torsion-free Jónsson modules and show that every Jónsson module is torsion or torsion-free, generalizing a result of Gilmer and Heinzer. We show that Jónsson modules whose cardinality is greater than that of the operator ring cannot be proved to exist in ZFC, and give several necessary and sufficient conditions for a Jónsson module to be countable. Time-permitting, we provide a few applications of our results. (Received September 23, 2006)