Daniel C. Cohen, Graham Denham and Michael J. Falk* (michael.falk@nau.edu), Dept. of Mathematics and Statistics, Northern Arizona University, Flagstaff, AZ 86011-5717, and Alexander N. Varchenko. Resonant weights and critical loci of rational functions. Preliminary report.

Suppose $\alpha_1, \ldots, \alpha_n$ are linear functionals on \mathbb{C}^ℓ and $\lambda \in (\mathbb{C}^*)^n$. We study the critical locus Σ_λ of the function $\Phi = \prod_{i=1}^n \alpha_i^{\lambda_i}$ in relation to the \mathbb{C} -algebra A generated by the 1-forms $\omega_i := d\alpha_i/\alpha_i$. Conjecturally, if the p^{th} cohomology $H^p(A, \omega_\lambda)$ of A relative to multiplication by $\omega_\lambda = \sum_i \lambda_i \omega_i$ is nonzero, then Σ_λ has codimension at most p. We prove this in the case $\ell = 3$ using recent results of Falk and Yuzvinsky on multinets and pencils of curves. We extend the $\ell = 3$ result, in a weaker form, to arbitrary ℓ , in the special case where p is minimal with $H^p(A, \omega_\lambda) \neq 0$, and some nonzero element of $H^p(A, \omega_\lambda)$ is represented by a decomposable p-form. We indicate a potential alternate approach to these results via "tropical implicitization."

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