We discuss some challenging graph theory problems having roots in (or immediate connections to) the combinatorics of finite partially ordered sets. Featured will be: (1) The Middle Two Levels conjecture — the bipartite graph formed by the subsets of size $n$ and $n + 1$ of a $2n + 1$-element set is hamiltonian; (2) The Monotone Hamiltonian path problem — in the subset lattice, there is a hamiltonian path starting at the empty set so that when a set $A$ is visited, the path has already visited every subset of $A$, with at most one exception. We will also discuss problems involving posets associated with planar graphs and issues of planarity for diagrams of posets. (Received September 17, 2011)