Quasi-residual and quasi-derived designs are defined to be 2-designs with the parameters of residual and derived designs of a symmetric design. If a quasi-residual (resp. a quasi-derived) design is in fact a residual (resp. a derived) design of a symmetric design, then it is called embeddable. Otherwise, it is said to be non-embeddable. The embedding problem of quasi-residual and quasi-derived design into a symmetric design is an old and natural question. Bhattacharya (1944) gave the first example of quasi-residual design which is non-embeddable. We consider quasi-residual and quasi-derived designs with parameters corresponding to residual and derived designs of Hadamard designs. Such designs have parameters $2 - (2k, k, k - 1)$ and $2 - (2k - 1, k - 1, k - 2)$ respectively. We refer to designs with such parameters as quasi-residual and quasi-derived Hadamard designs of order $k$. We give some results on such designs of small orders. We then use recursive methods in order to obtain infinite families of non-embeddable quasi-residual and quasi-derived Hadamard designs. (Received September 21, 2011)