Embeddability of infinite graphs.

Robertson and Seymour proved that there is a function \( f(g) \) tending to infinity so that, if a graph \( G \) does not embed in any surface of Euler characteristic at least 2 – 2\( g \), then \( G \) has one of the following as a minor:

1. \( f(g) \) disjoint copies of either \( K_{3,3} \) or \( K_5 \);
2. \( f(g) \) copies of either \( K_{3,3} \) or \( K_5 \) that are disjoint except for a common vertex;
3. \( f(g) \) copies of either \( K_{3,3} \) or \( K_5 \) that are disjoint except for two common vertices; or
4. \( K_{3,f(g)} \).

We have proved the following extension to infinite graphs.

**Theorem.** A countable graph \( G \) embeds in some orientable surface if and only if \( G \) does not contain as a minor one of the following graphs:

1. infinitely many disjoint copies of either \( K_{3,3} \) or \( K_5 \);
2. infinitely many copies of either \( K_{3,3} \) or \( K_5 \) that are disjoint except for a common vertex;
3. infinitely many copies of either \( K_{3,3} \) or \( K_5 \) that are disjoint except for two common vertices; or
4. \( K_{3,\infty} \).

(Received September 21, 2011)