Generalized permutohedra are the polytopes obtained from the permutohedron by changing the edge lengths while preserving the edge directions, possibly identifying vertices along the way. We introduce a “lifting” construction for these polytopes. We show how this construction gives rise to Stasheff’s multiplihedron from homotopy theory, and to the more general “nestomultiplihedra”, answering a question of Devadoss and Forcey.

We construct a subdivision of any lifted generalized permutohedron whose pieces are indexed by compositions. The volume of each piece is a polynomial, whose combinatorial properties we investigate. We show how this “composition polynomial” arises naturally in the polynomial interpolation of an exponential function. We prove that its coefficients are positive integers, and we conjecture that they are unimodal. (Received September 22, 2011)