A bisection of a graph is a bipartition of its vertex set in which the number of vertices in the two parts differ by at most 1, and its size is the number of edges which go across the two parts. Motivated by several questions and conjectures of Bollobás and Scott, we study maximum bisections of graphs. First, we extend the classical Edwards bound on maximum cuts to bisections. A simple corollary of our result implies that every graph on $n$ vertices and $m$ edges with no isolated vertices, and maximum degree at most $n/3 + 1$, admits a bisection of size at least $m/2 + n/6$. Then using the tools that we developed to extend Edwards’s bound, we prove a judicious bisection result which states that graphs with large minimum degree have a bisection in which both parts span relatively few edges. A special case of this general theorem answers a conjecture of Bollobás and Scott, and shows that every graph on $n$ vertices and $m$ edges of minimum degree at least 2 admits a bisection in which the number of edges in each part is at most $(1/3 + o(1))m$. We also present several other results on bisections of graphs.

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