Consecutive Matches in Permutations and cycle structures of permutations.

Let $\tau \in S_m$. A permutation $\sigma = \sigma_1 \ldots \sigma_n \in S_n$ has a $\tau$-match at position $i$ if $\sigma_i \ldots \sigma_{i+m-1}$ has the same relative order as $\tau$. A cycle $C = (\sigma_0, \ldots, \sigma_{n-1}) \in S_n$ has a cycle-$\tau$-match at position $i$ if $\sigma_i \ldots \sigma_{i+m-1}$ has the same relative order as $\tau$ with the subscripts taken mod $n$. Let $N\mathcal{M}_n(\tau)$ be the set of all $\sigma$ in $S_n$ that have no $\tau$-matches. Let $N\mathcal{CM}_n(\tau)$ be the set of all $\sigma$ in $S_n$ that have no cycle-$\tau$-matches within any of the cycles.

Consider the generating functions

$$NM_{\tau}(t, y, x) = \sum_{n \geq 0} \frac{t^n}{n!} \sum_{\sigma \in N\mathcal{M}_n(\tau)} y^{1 + \text{des}(\sigma)} x^{\text{Lmin}(\sigma)}$$

$$N\mathcal{CM}_{\tau}(t, y, x) = \sum_{n \geq 0} \frac{t^n}{n!} \sum_{\sigma \in N\mathcal{CM}_n(\tau)} y^{\text{cdes}(\sigma)} x^{\text{cyc}(\sigma)}.$$ 

For $\sigma \in S_n$, des($\sigma$) is the number of descents, Lmin($\sigma$) is the number of left-to-right minima, cdes($\sigma$) is the number of descents of each cycle, and cyc($\sigma$) is the number of cycles.

We discuss why these generating functions are equal when $\tau$ starts with 1 and give some results for families of patterns that start with 1. (Received September 22, 2011)