In this paper we are settling a long-standing open problem. We prove that it is NP-hard to recognize T-tenacious graphs for any fixed positive rational number \( T \).

The concept of tenacity of a graph \( G \) was introduced by Cozzens, Moazzami and Stueckel in 1992, as a useful measure of the "vulnerability" of \( G \). The tenacity of a graph \( G \), \( T(G) \), is defined by \( T(G) = \min\{\frac{|S|+\tau(G-S)}{\omega(G-S)}\} \), where the minimum is taken over all vertex cutsets \( S \) of \( G \). We define \( \tau(G-S) \) to be the number of vertices in the largest component of the graph \( G - S \), and \( \omega(G-S) \) the number of components of \( G - S \). A connected graph \( G \) is called T-tenacious if \( |S| + \tau(G-S) \geq T\omega(G-S) \) holds for any subset \( S \) of vertices of \( G \) with \( \omega(G-S) > 1 \). If \( G \) is not complete, then there is a largest \( T \) such that \( G \) is T-tenacious; this \( T \) is the tenacity of \( G \). On the other hand, a complete graph contains no vertex cutset and so it is T-tenacious for every \( T \).

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