In this work we will work in finite fields $\mathbb{F}_{q^2}$ where $q = p^n$ for some prime $p$. We will work with hyperelliptic curves denoted $Y_S: y^2 = f_S(x) = \prod_{s \in S}(x - s)$ where $S \subset \mathbb{F}_{q^2}$. $X_S$ represents the non-singular projective model of $Y_S$. We utilize the Hasse-Weil bound $|X_S(\mathbb{F}_{q^2}) - (q^2 + 1)| \leq 2gq$ and $\Lambda(S) = \sum_{b \in \mathbb{F}_{q^2}} \chi(f_S(b)) = |Q_S| - |Q'_S|$, as we try to determine when $X_S$ is maximal, minimal and/or optimal. We say $X_S$ is maximal if the upper Hasse-Weil bound is achieved and if

$$|X_S(\mathbb{F}_{q^2})| = \begin{cases} 1 + q^2 + q(|S| - 1) & \text{if } |S| \text{ is odd}, \\ 1 + q^2 + q(|S| - 2) & \text{if } |S| \text{ is even}. \end{cases}$$

We say $X_S$ is minimal if the lower Hasse-Weil bound is reached and if

$$|X_S(\mathbb{F}_{q^2})| = \begin{cases} 1 + q^2 - q(|S| - 1) & \text{if } |S| \text{ is odd}, \\ 1 + q^2 - q(|S| - 2) & \text{if } |S| \text{ is even}. \end{cases}$$

Finally we say $X_S$ is optimal if $\Lambda(S) = q^2 - |S|$. There is an overlap of maximal curves and minimal curves; there is also an overlap between maximal curves and optimal curves. In particular maximal sets, $S$, and their associated maximal curves $X_S$ are useful in coding theory. (Received September 20, 2011)