We study the Mordell-Weil groups of Jacobians of curves defined over rational function fields of positive characteristic. Under certain conditions, we show that the rank of the Mordell-Weil group can be arbitrarily large by using Artin-Schreier theory to study the order of vanishing of L-functions. In other situations, we give a formula for the rank in terms of the endomorphism ring of a companion Jacobian. The proof uses an Artin-Schreier variant of Berger’s construction of surfaces dominated by a product of curves in towers. (Received September 21, 2011)