Oguiso and Shioda described the possible Mordell-Weil lattices that could arise for rational elliptic surfaces over \( \mathbb{Q} \) (equivalently, the Mordell-Weil group of an elliptic curve over a rational function field in one variable). The study of specific Mordell-Weil lattices has connections to invariant theory and inverse Galois theory (for instance, of some Weyl groups of root lattices), and Shioda has used these techniques to construct “excellent” families of rational elliptic surfaces with additive reduction and Mordell-Weil lattice \( E_8, E_7, E_6, D_4 \) etc, and also for multiplicative reduction and Mordell-Weil lattice \( E_6 \).

We describe joint work with Shioda which deals with the multiplicative reduction case with Mordell-Weil lattice \( E_8 \) or \( E_7 \). The parameters of the “excellent” families are related to the fundamental multiplicative invariants of the corresponding Weyl groups. We use our results to produce examples of elliptic surfaces for which the splitting field has large Galois group, and also examples for which it has trivial Galois group (all sections defined over \( \mathbb{Q} \)). (Received September 22, 2011)