Affine fibrations and Vénéréau-type polynomials.

The Vénéréau polynomials $f_n = y + x^n(xz + y(yu + z^2))$ ($n \geq 1$) are well known examples of polynomials with many coordinate-like properties, including that they are coordinates when $x$ is inverted, or upon going modulo $x$. $f_n$ is known to be a $\mathbb{C}[x]$-coordinate for $n \geq 2$, but the question remains open for $n = 1$.

We show that many polynomials with this coordinate-like property are in fact coordinates over $\mathbb{C}[x]$. In the case of $\mathbb{C}^4$, the additional assumption that a polynomial becomes a coordinate of a tame automorphism upon inverting $x$ is sufficient to guarantee it is a $\mathbb{C}[x]$-coordinate. As corollaries, we obtain that Vénéréau-type polynomials are all 1-stable $\mathbb{C}[x]$-coordinates, and characterize them as the simplest potential counterexamples to the Dolgachev-Weisfeiler conjecture. (Received September 20, 2011)