Polynomials that are sums of squares (SOS) can be efficiently optimized via semidefinite programming. In this paper, we investigate when a nonnegative polynomial $p \in \mathbb{R}[x_1, \ldots, x_n]$ is SOS. For nonnegative polynomials of fixed degree, previous results by Blekherman show that, as $n \to \infty$, the fraction of nonnegative polynomials that are SOS approaches zero. However, these bounds are loose, and this fraction is unknown for most polynomials of low degree in few variables. Our research focuses on estimating this fraction for nonnegative bivariate polynomials of degree at most four in each variable. The fraction of nonnegative polynomials that are SOS can be estimated as the ratio of volumes of two naturally definable convex bodies of dimension 24. To avoid computing these volumes directly, we implemented a version of Smith’s rapidly mixing hit and run technique for uniform sampling from a convex body. (Received August 19, 2011)