We show that the quantum coordinate ring of the unipotent subgroup $N(w)$ of a symmetric Kac-Moody group $G$ associated with a Weyl group element $w$ has the structure of a quantum cluster algebra. This quantum cluster structure arises naturally from a subcategory $C_w$ of the module category of the corresponding preprojective algebra. An important ingredient of the proof is a system of quantum determinantal identities which can be viewed as a $q$-analogue of a T-system. In case $G$ is a simple algebraic group of type $A$, $D$, $E$, we deduce from these results that the quantum coordinate ring of an open cell of a partial flag variety attached to $G$ also has a cluster structure. (Received September 21, 2011)