We define the stable derived category of any ring $R$ as the homotopy category of a model category structure on the category of chain complexes of $R$-modules. The construction works for any ring $R$ but recovers the stable derived category introduced by Krause in the case that $R$ is Noetherian. A dual construction also using model categories is studied by Jim Gillespie. The model category approach allows us to study this category more transparently and to develop certain computational aspects. In particular, taking $R$ to be the ring of polynomials in two variables over a field modulo the ideal generated by the homogenous polynomials in degree two, we are able to exhibit several non trivial objects of the stable derived category and maps between them, giving us new insights into the stable derived category of this ring. (Received September 20, 2011)