Two major tools in representation theory are:

1) restricting representations to compact subgroups since the representation theory of compact groups is well understood (this leads to the category of Harish-Chandra modules) and

2) exploiting joint eigenspaces of a Cartan (weight theory, which leads to Category $\mathcal{O}$).

Motivated by combining the theory of compact groups with highest weight theory, we define mixed subgroups. The category of $(\mathfrak{g}, M)$ modules, where $M$ is a mixed subgroup, generalizes both Category $\mathcal{O}$ and the category of Harish-Chandra modules: they can be recovered by choosing $M$ appropriately.

We classify the irreducibles in $C(\mathfrak{g}, M)$. We relate $K\backslash G/B$, $M\backslash G/B$, and discuss the Bruhat order. Finally, we show certain equivalence classes of mixed subgroup orbits on the flag variety to be in bijection with mixed subgroup orbits on flag varieties for smaller generalized Harish-Chandra pairs: this permits a generalization of the common technique of using root subsystems to simplify Kazhdan-Lusztig-Vogan polynomial computations.

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