An analytic function without radial boundary values.

It is a fairly well known, but not immediately accessible, fact that there exists a function analytic on $D = \{ z \in \mathbb{C} : |z| < 1 \}$ that has a radial limit at no point of the boundary $\{ z \in \mathbb{C} : |z| = 1 \}$. The function $f(z) = \sum_{n=0}^{\infty} z^n$ can easily be shown to be an example of such a function. The very short proof of this is buried in the paper of J. M. Ash and M. T. Karaev, “On the boundary behavior of special classes of C-functions and analytic functions,” which will appear in the International Mathematical Forum. Since I think this is a useful fact, I will sketch the proof that the analytic on $D$ function $f(z)$ is radially divergent on the entire boundary of $D$. (Received September 21, 2011)