Let $G$ be a connected semisimple Lie group with finite center. In 1969, Helgason and Johnson determined an explicit parametrization of the set of spherical functions on $G$ which are bounded. This result has several important consequences on the $L^1$ spherical harmonic analysis on $G$.

By restriction to a Cartan subspace, the spherical functions on $G$ can be considered as special instances of the hypergeometric functions associated with root systems. This class of (generally multivariate) hypergeometric functions has been introduced by Heckman and Opdam in the late 1980s, with important contributions by Cherednik and by Opdam in the middle 1990s and, more recently, by Schapira in 2008. In this talk, we extend the result of Helgason and Johnson and find a parametrization of the hypergeometric functions associated with root systems which are bounded. Some applications are presented. (Received September 10, 2011)