The field of nonlinear dispersive evolution equations has undergone rapid changes in recent years. The equations in question are Hamiltonian and encompass a wide class ranging from nonlinear Klein-Gordon, wave, and Schroedinger equations on the one hand, to more geometric equations such as wave maps and other so-called 'field equations' of physics on the other hand. These equations have traditionally been studied from the point of view of the fundamental well-posedness problem locally in time which often requires large amounts of analytical machinery. The question of global-in-time properties of the evolution is the subject of much ongoing research in nonlinear evolution equations. Within the past five to six years, several open problems have been settled in the field by introducing new ideas such as concentration-compactness for evolution equations, and the use of invariant manifolds from hyperbolic dynamics. We shall give an overview over some of these developments. The author's work is joint with Kenji Nakanishi from Kyoto University, Japan, and Joachim Krieger from EPFL, Switzerland. (Received September 18, 2011)