
We consider regularized approximate cloaking for the Helmholtz equation. Various cloaking schemes have been recently proposed and extensively investigated. The existing cloaking schemes in literature are (optimally) within $|\ln \rho|^{-1}$ in 2D and $\rho$ in 3D of the perfect cloaking, where $\rho$ denotes the regularization parameter. In this work, we develop a cloaking scheme with a well-designed lossy layer right outside the cloaked region that can produce significantly enhanced near-cloaking performance. In fact, it is proved that the proposed cloaking scheme could (optimally) achieve $\rho^N$ in $\mathbb{R}^N$, $N \geq 2$, within the perfect cloaking. It is also shown that the limit of the proposed lossy layer corresponds to a sound-hard layer. We work with general geometry and arbitrary cloaked contents of the proposed cloaking device. Numerical examples are given to demonstrate the sharpness of our estimates. (Received September 22, 2011)