The advent of transformation optics and metamaterials has made possible devices producing extreme effects on wave propagation. We will describe a class of invisible reservoirs and amplifiers for waves, which we refer to as Schrödinger hats. The unifying principle on which these are based admits such devices for any wave phenomenon modeled by either the Helmholtz or Schrödinger equation, e.g., polarized waves in electromagnetism, acoustical waves and matter waves in quantum mechanics. Schrödinger hats occupy one part of a parameter-space continuum of wave-manipulating structures which also contains standard transformation optics based cloaks, resonant cloaks and cloaked sensors. We discuss their properties and some possible implementations. (Received September 08, 2011)