Consider the Schrödinger operator $H = -\Delta + V$ on $\mathbb{R}^2$ and $P_{ac}(H)$ the projection onto the absolutely continuous spectrum of $H$. We prove $L^1(\mathbb{R}^2) \to L^\infty(\mathbb{R}^2)$ estimates for the evolution $e^{itH}P_{ac}(H)$ when there are obstructions, resonances and/or an eigenvalue of $H$ at zero energy. In particular, we show that the existence of a mild resonance of $H$ at zero energy does not destroy the $t^{-1}$ decay rate. We also show that the existence of a more singular resonance or eigenvalue at zero energy destroys the decay rate, but does lead to a bounded evolution. (Received July 15, 2011)