Noncommutative Real Ideals and an Algorithm for Computing Them.

The zero set of a noncommutative polynomial $p$ is the set of all pairs $(X, v)$, where $X$ is a tuple of square matrices and $v$ is a vector, such that $p(X)v = 0$. If $p(X)v = 0$ whenever $q(X)v = 0$ for some other noncommutative polynomial $q$, then the zero set of $q$ is contained in the zero set of $p$. A polynomial $q$ has the left nullstellensatz property if whenever the zero set of a polynomial $p$ contains the zero set of $q$, the polynomial $p$ is equal to $fq$ for some polynomial $f$. I give a framework for proving that certain polynomials have this left nullstellensatz property. Using this adds several natural cases to those previously known. Further, I introduce a noncommutative analog to the concept of a real ideal. The left ideal generated by a noncommutative polynomial $q$ with the left nullstellensatz property must be a real ideal. I provide an algorithm for computing the smallest noncommutative real left ideal containing a noncommutative polynomial $q$. (Received September 21, 2011)