If $\phi$ is an analytic map of the disk into itself and $H$ is a Hilbert space of analytic functions on the disk, the composition operator $C_\phi$ is the operator given by $C_\phi f = f \circ \phi$ for $f$ in $H$.

Nordgren, Rosenthal, and Wintrobe (1984) observed that, if $\phi$ is a hyperbolic automorphism of the disk, then $C_\phi^* - I$ acting on the Hardy space is a ‘universal operator’ in the sense that every bounded operator on a Hilbert space is unitarily equivalent to a restriction of a multiple of this operator to an invariant subspace. This incomplete survey of results on invariant subspaces of composition operators will suggest that this is a rich area for future study.

The most striking result up to now has been the characterization by Montes, Ponce, and Shkarin (2010) of the lattice of invariant subspaces of a composition operator with symbol a linear fractional map of the disk into (but not onto) itself.

Much work, a promising beginning, concerns invariant subspaces of composition operators that are also invariant for the operator of multiplication by $z$. Work for Hermitian weighted composition operators has led to identifying extremal functions for subspaces in weighted Bergman spaces associated with the usual atomic inner functions. (Received August 17, 2011)