Let $\mathcal{S}$ denote the set of functions $b$, holomorphic in the unit ball of $\mathbb{C}^d$, such that the kernel $(1 - b(z) \overline{b(w)})(1 - \langle z, w \rangle)^{-1}$ is positive, and write $\mathcal{H}(b)$ for the corresponding reproducing kernel Hilbert space. In one variable these are known as the de Branges-Rovnyak spaces. Their theory is well-developed; the central objects are the backward shift operator on $\mathcal{H}(b)$ and the Aleksandrov-Clark (AC) measure $\mu$.

The natural analog of the AC measure in the multivariable setting is a certain positive linear functional on a (non-commutative) operator system. The next difficulty is to understand what should be meant by “backward shift.” We introduce a canonical solution to the Gleason problem in $\mathcal{H}(b)$ which preserves many features of the backward shift in the one-variable setting, and identify a subclass of $\mathcal{S}$ called quasi-extreme functions. (In one variable, these are the extreme points of the unit ball of $H^\infty$.) As an application we obtain a version of Clark’s theorem on rank-one perturbations of the backward shift, and some further function-theoretic results. (For example, $\mathcal{H}(b)$ is $z_j$-invariant for each $j = 1, \ldots, d$ if and only if $b$ is not quasi-extreme.) (Received September 07, 2011)