In this talk we discuss an abstract constrained optimization problem which appears commonly in the optimal control and inverse problem of linear partial differential equations. The main emphasis of the present study is on the case when the ordering cone for the optimization problem has an empty interior. To circumvent this major difficulty, we propose a new conical regularization approach in which the main idea is to replace the ordering cone by a family of dilating cones. We showed that this approach leads to a family of optimization problems that admit regular multipliers. Detailed convergence analysis is given. One of the main advantages of the proposed approach is that it is amenable for numerical computations. The motivation for the conical regularization is to overcome the difficulties associated to the lack of Slater’s type constrain qualification which is a common hurdle in numerous branches of applied mathematics including optimal control, inverse problems, vector optimization, set-valued optimization, sensitivity analysis, variational inequalities, among others. The approach remains valid in the setting of general Hilbert spaces and it does not require any sort of compactness or positivity condition on the operators involved. (Received September 21, 2011)