Applying the calculus of functors to the study of knot spaces gives spectral sequences, due to Sinha, converging to the homology and cohomology. This framework does not immediately lead to representatives of cycles and cocycles. By resolving knots with $k$ double points, Cattaneo, Cotta-Ramusino and Longoni produced explicit, non-trivial $k(d-3)$-dimensional cycles. We generalize these results to resolutions of singular knots with triple points, producing a non-trivial $3(d-8)$-dimensional cycle. This extends and corrects the results in a preprint of Longoni. The techniques we use are closely related to the combinatorics of the embedding calculus homology spectral sequence, suggesting that they may lead to recipes for geometric representatives for all of the cycles in that spectral sequence. (Received September 17, 2011)