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Daniel S. Freed* (dafr@math.utexas.edu), Department of Mathematics, University of Texas, 1 University Station C1200, Austin, TX 78712-0257. *The cobordism hypothesis: quantum field theory + homotopy invariance = higher algebra.*

Quantum field theory, which physicists initiated in the 1920s to construct a quantum theory of the electromagnetic field, has long attracted interest in mathematics. Over the past 30 years the geometric side of quantum field theory has come to the fore. In 1988 Witten introduced *topological* quantum field theory (TQFT) as a home for topological invariants of Donaldson and Jones. TQFT quickly became an inspiration for invariants in low-dimensional topology as well as a subject for mathematical study.

In the 1990s Baez-Dolan formulated a “cobordism hypothesis” characterizing TQFTs which are fully extended: n -dimensional theories which include invariants for manifolds of all dimensions $\leq n$. This was put into a rigorous context and proved first for $n = 2$ by Hopkins and Lurie, then for all dimensions n by Lurie. Deep ideas in Morse theory and higher categories are central to the proof; no physics is necessary. Rather, just as standard homology uses algebra to study topological spaces, TQFT uses algebra to study smooth manifolds. These ideas are now finding applications in other parts of topology and algebra, for example in representation theory.

In the lecture I will explain some of the basic ideas and examples, and give some hints about the proof. (Received September 16, 2011)