Legendrian knot theory is the study of knots and links which satisfy a geometric condition imposed by a contact structure. In recent years, generating families and Morse theory have been used to develop new homological invariants for knots and links in $\mathbb{R}^3$ and $S^1 \times \mathbb{R}^2$. These invariants parallel Legendrian contact homology, and can be used to show that certain knots and links are topologically equivalent but not Legendrian equivalent. This talk will explore the extension of the generating family approach to circle valued functions, allowing the study of Legendrian knots and links in $T^2 \times \mathbb{R}$. Using techniques of Morse-Novikov theory, it is possible to define homological invariants which demonstrate that the components of certain links cannot be interchanged by Legendrian isotopy, although such an interchange is possible under topological isotopy. (Received September 13, 2011)