We present an algorithm to test permutational isomorphism of permutation groups in time polynomial in the order of the groups and simply exponential in the degree. In the case of transitive groups we in fact list all permutational isomorphisms within the stated time bound. The algorithm involves an analysis of a structure tree (imprimitivity hierarchy), estimates for primitive groups, and special handling of the case when the alternating or symmetric group acts at a node of the tree. The general case reduces to the transitive case via the “twisted code equivalence” problem. A code of length $n$ is a set of strings of length $n$ over a finite alphabet. An equivalence of codes $A$ and $B$ is a permutation $\pi \in S_n$ such that $A^\pi = B$ (permuting the positions). Twisted code equivalence additionally allows a group action on the alphabet. Our solution to this problem generalizes Luks’s dynamic programming algorithm for hypergraph isomorphism.

The transitive case of our result is a key ingredient in our polynomial-time isomorphism test for semisimple groups (groups with no non-trivial abelian normal subgroups), given by their Cayley tables. The best previous bound for this class was $n^{\log \log n}$ by the present authors and J. Grochow. (Received September 20, 2011)