We discuss the well-posedness of a nonlinear PDE system that describes boundary layers in complex (non-Newtonian) fluids. The boundary layer theory for Newtonian fluids, based on Navier-Stokes equations, was developed by Prandtl more than one hundred years ago, addressing the existence of a thin velocity transition layer next to a solid boundary. Boundary layers arise in many applications in aerodynamics and fluid dynamics, but there is still a lack of fundamental understanding of the corresponding boundary layer equations, namely Prandtl’s system. Indeed, it has recently been shown that these equations are not necessarily well-posed.

Fluids with complex microstructure, such as polymers, suspensions, and granular materials, abound in biological science, materials sciences as well as in many industrial processes. In the limit of high elasticity (Weissenberg number), a boundary layer problem similar to Prandtl’s arises. We derive the system of equations to describe the behavior near the boundary. By taking advantage of the Lagrangian coordinates, we show that the non-Newtonian boundary layer system is actually well-posed. (Received September 21, 2011)