The smallest enclosing circle problem asks for the circle of smallest radius enclosing a given set of finite points on the plane. This problem was introduced in the 19th century by James Joseph Sylvester (1814–1897). After more than a century, extended work on the problem remains very active. In this talk we present new results on the following two problems: the smallest enclosing ball problem and the smallest intersecting ball problem. The smallest enclosing ball problem states that given a finite number of nonempty closed subsets of a Banach space, find a ball with the smallest radius that encloses all of the sets. Similarly, the smallest intersecting ball problem finds a ball with the smallest radius that intersects all of the sets. We will focus on the existence and uniqueness of optimal solutions, as well as, necessary and sufficient optimality conditions for the problems. This work is a continuation of our effort in shedding new light to classical geometry problems using advanced tools of variational analysis and optimization. (Received September 22, 2011)