Artem S Novozhilov* (anovozhilov@gmail.com), Vladimir P Posvyanskii and Alexander S Bratus. Reaction-diffusion replicator equation: Stability and asymptotic behavior.

The replicator equation is known to provide a general modeling framework for several distinct areas in mathematical biology. There are several different approaches to add space to the replicator equation. As a counterpart of the local model we consider the model

$$\frac{\partial u_i}{\partial t} = u_i [(Au)_i - f^{sp}(t)] + d_i \Delta u_i, \quad i = 1, \ldots, n,$$

where now $u = u(x, t), \ x \in \Omega \subset \mathbb{R}^k, \ k = 1, 2, 3, \ d_i > 0$ are diffusion coefficients, and the mean integral fitness is given by $f^{sp}(t) = \int_{\Omega} (Au, u) \, dx$. This approach allows analytical investigation of (1): the tool which was mainly missing in the analysis of replicator equations with explicit space. In our work, we show that for some values of the diffusion coefficients spatially heterogeneous solutions appear. Using a definition for the stability in the mean integral sense we prove that these heterogeneous solutions can be attracting; in particular this is the case for Eigen’s hypercycle. Defining in some natural way evolutionary stable states for the distributed system (1), we provide the conditions for this distributed state to be an asymptotically stable stationary solution to (1). (Received September 13, 2011)