In the absence of decoherence, the dynamics of a controlled quantum system is given by a Schrödinger equation, \( x' = Ax + u(t)Bx \), where \( x \) lies in some infinite dimensional Hilbert space, \( A \) is a skew-adjoint operator, \( B \) is a skew-symmetric linear operator accounting for the interaction of the environment with the system (e.g., through a laser) and \( u \) is the time variable scalar intensity of the control. We will restrict ourselves to the case where \( A \) has a purely discrete spectrum. The energy of the system is the \( A^{1/2} \) norm of \( x \).

A bilinear system is weakly coupled if \( |\Im \langle Ax, Bx \rangle| \leq |\langle Ax, x \rangle| \) for every \( x \). Most of the physical examples have this feature. For weakly-coupled bilinear systems, there exists an a priori bound for the growth of energy of the system in terms of the \( L^1 \) norm of the control \( u \). In particular, such systems can be approximated with arbitrary precision by their finite dimensional Galerkin approximations. This gives a theoretical justification of the approximations usually done in practice and provides constructive control algorithms.

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