Due to their applications in network coding, public-key cryptography, and space-time coding, both rank-metric codes and matrix codes, also known as array codes and space-time codes over finite fields, have garnered significant attention. We focus on characterizing rank-metric and matrix codes that are both efficient, i.e. have high dimension, and effective at error correction, i.e. have high minimum distance. A number of researchers have contributed to the foundation of duality theory for rank-metric and matrix codes, which has demonstrated that the inherent trade-off between dimension and minimum distance for a code is reversed for its dual code; specifically, if a code has high dimension and low minimum distance, then its dual code will have low dimension and high minimum distance. Thus, with an aim towards finding codes with a perfectly balanced trade-off, we study self-dual matrix codes. In particular, we enumerate the equivalence classes of self-dual matrix codes of short lengths over small finite fields. To perform this classification, we also examine the notion of equivalence for rank-metric and matrix codes and use this to characterize the automorphism groups of these codes. (Received August 16, 2011)